

The universal bound property for a class of second order ODEs

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Abstract: We consider the scalar second order ODE

$$u'' + |u'|^\alpha |u'| + |u|^\beta u = 0,$$

where α, β are two positive numbers, after the limited elementary approach of [3], Philippe Souplet [5] gave a definitive negative answer at least when $\alpha \geq \beta \geq 0$. Strangely enough, although this equation has been studied very carefully in [4] which several extensions in [1] and [2], it seems that nobody questioned the possibility of universal boundedness when $0 < \alpha < \beta$ and the non-linear semi-group $S(t)$ generated on \mathbb{R}^2 by the system in (u, u') . We prove that $S(t)\mathbb{R}^2$ is bounded for all $t > 0$ whenever $0 < \alpha < \beta$ and moreover there is a constant C independent of the initial data such that

$$\forall t > 0, \quad u'^2(t) + |u|^{\beta+2} \leq C \max\left\{t^{-\frac{2}{\alpha}}, t^{-\frac{(\alpha+1)(\beta+2)}{\beta-\alpha}}\right\}.$$

Key word: Second order scalar ODE, Decay rate.

Références

- [1] M. ABDELLI AND A. HARAUX, (2014), *Global behavior of the solutions to a class of nonlinear second order ODE's*, Nonlinear Analysis , **96**, 18-73. 95, 297-321.
- [2] M. ABDELLI, M. ANGUIANO AND A. HARAUX, (2011), *Existence, uniqueness and global behavior of the solutions to some nonlinear vector equations in a finite dimensional Hilbert space*, Nonlinear Analysis, **161** (2017), 157-181.
- [3] A. HARAUX, *Remarks on the wave equation with a nonlinear term with respect to the velocity*, Portugal. Math. **49** (1992), 557-454.
- [4] A. HARAUX , *Sharp decay estimates of the solutions to a class of nonlinear second order ODE's* J. Differential Equations **9**, (2011), 149-69.
- [5] P. SOUPLET, *Critical exponents, special large-time behavior and oscillatory blow-up in nonlinear ODE's*, Differential Integral Equations **11** (1998), 147-167.