

## The universal bound property for a class of second order ODEs

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**Abstract:** We consider the scalar second order ODE

$$u'' + |u'|^\alpha |u'| + |u|^\beta u = 0,$$

where  $\alpha, \beta$  are two positive numbers, after the limited elementary approach of [3], Philippe Souplet [5] gave a definitive negative answer at least when  $\alpha \geq \beta \geq 0$ . Strangely enough, although this equation has been studied very carefully in [4] which several extensions in [1] and [2], it seems that nobody questioned the possibility of universal boundedness when  $0 < \alpha < \beta$  and the non-linear semi-group  $S(t)$  generated on  $\mathbb{R}^2$  by the system in  $(u, u')$ . We prove that  $S(t)\mathbb{R}^2$  is bounded for all  $t > 0$  whenever  $0 < \alpha < \beta$  and moreover there is a constant  $C$  independent of the initial data such that

$$\forall t > 0, \quad u'^2(t) + |u|^{\beta+2} \leq C \max\left\{t^{-\frac{2}{\alpha}}, t^{-\frac{(\alpha+1)(\beta+2)}{\beta-\alpha}}\right\}.$$

**Key word:** Second order scalar ODE, Decay rate.

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