Global persistence of geometric structures for stratified Euler system with fractional dissipation

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The present work deals with the stratified Euler system with fractional dissipation,

$$\begin{cases} \partial_t v + v\cdot\nabla v + \nabla p = \rho e_2, \quad \partial_t \rho + v\cdot\nabla\rho + (-\Delta)^{\frac{\alpha}{2}}\rho = 0 \quad (t,x)\in\mathbb{R}_+\times\mathbb{R}^2, \\ \operatorname{div} v = 0, \quad (v,\rho)_{|t=0} = (v_0,\rho_0). \end{cases} \tag{1}$$

Here, v refers to the velocity vector field in $\mathbb{R}^2$, the pressure p and the density ρ are two scalar functions. The buoyancy force ρe_2 models the effects of the gravity with $e_2 = (0,1)$ and $(-\Delta)^{\frac{\alpha}{2}}$ designates the fractional laplacian with $\alpha \in]0,2]$. In $\mathbb{R}^2$, the vorticity ω of the velocity field v may be identified by $\omega \triangleq \partial_1 v^2 - \partial_2 v^1$. Consequently, the vorticity-density formulation of (1) is given by

$$\begin{cases} \partial_t \omega + v\cdot\nabla\omega = \partial_1\rho, \quad \partial_t \rho + v\cdot\nabla\rho + (-\Delta)^{\frac{\alpha}{2}}\rho = 0 \quad (t,x)\in\mathbb{R}_+\times\mathbb{R}^2, \\ (\omega,\rho)_{|t=0} = (\omega_0,\rho_0). \end{cases} \tag{2}$$

If $\rho \equiv \rho_0$, the system (1) is reduced to the classical $2d-$ incompressible Euler equations which are well-studied. In particular, when the initial vorticity having a vortex patch structure, that is to say, ω_0 is uniformly distributed over a bounde domain Ω_0 in the sense $\omega_0 = \mathbf{1}_{\Omega_0}$, Chemin has succeed in [2] to recover the Euler system globally in time. The cornerstone in his proof is the stationary logarithmic estimate. Afterwards, the Chemin's result was extended for several systems and differents regularities by various authors, see for instance [1, 3, 4, 5, 6, 7, 8, 9].

The study of the vortex patch problem for (2) has been started recently in [9] where Hmidi and the author studied the case $\alpha = 2$ and showed if the boundary of $\partial\Omega_0$ is a Jordan curve of $C^{1+\varepsilon}$ regularity with $\varepsilon \in]0,1[$ then the velocity is Lipschitz for any positive time and the advected domain $\Omega_t \triangleq \Psi(t,\Omega_0)$ keeps its initial regularity. Moreover, we show that the vorticity $\omega(t) = \mathbf{1}_{\Omega_t} + \widetilde{\rho}(t)$, where $\widetilde{\rho}$ is a smooth function.

Here, we intend to lead the same result for the critical case $\alpha = 1$. Roughly speaking, we shall prove the persistence regularity of the initial patch and study the asymptotic behavior for the density.

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