Identification and Control
Some new challenges

18-20 June 2019

Hotel Monastir Center, Monastir, Tunisia

Program and abstracts

UR Analysis and Control of PDEs, University of Monastir, Tunisia

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Program

Tuesday 18 June 2019

8h00-8h45 - ***Registration***
8h45-9h00 - ***Welcome***

Session Chairman : Makram Hamouda

9h-9h50 - E. Soccorsi: *Introduction to Borg-Levinson Inverse Spectral Theory*
9h50-10h40 - L. Tébou: *Simultaneous and indirect control of waves: some recent developments and open problems*
10h40-11h10 - ***Coffee break***
11h10-12h - Y. Kian: *Introduction au problème de Calderón*
12h-14h30 - ***Lunch***

Session Chairman : Éric Soccorsi

14h30-15h05 - M. Aouadi: *Exact controllability for nonlinear thermoviscoelastic plate problem*
15h05-15h40 - M.Hamouda: *Theoretical and numerical aspects related to the energy of some Timoshenko systems*
15h40-16h - ***Coffee break***
16h-16h20 - A.Benaissa: *A general decay result of a wave equation with a dynamic boundary control of diffusive type*
16h20-16h40 - M. Zerguine: *Global persistence of geometric structures for stratfied Euler system with fractional dissipation*
16h40-17h - A. Duca: *Permutating quantum eigenmodes by using the quasi-adiabatic motion of a wall*
17h-17h20 - M. Abdelli: *The universal bound property for a class of second order ODEs*
17h20-18h10 - ***Short talks and Posters presentation***

Wednesday 19 June 2019

Session Chairman : Manuel González-Burgos

9h-9h50 - Y. Kian: *Introduction au problème de Calderón*
9h50-10h40 - E. Soccorsi: *Introduction to Borg-Levinson Inverse Spectral Theory*
10h40-11h10 - ***Coffee break***
11h10-12h - L. Tébou: *Simultaneous and indirect control of waves: some recent developments and open problems*
12h-14h30 - ***Lunch***

Session Chairman : Yavar Kian

14h30-15h05 - S. K. Harouna: *A Stochastic Sub-grid Viscosity Model and Wavelet based Method for Images Assimilation*
15h05-15h40 - A. Bchatnia: *Observability and stabilizability of the wave equation with moving boundary*
15h40-16h - ***Coffee break***
16h-16h20 - Y. Hamzaoui: *Existence of solutions to Neumann problem involving multiple critical exponents*
16h20-16h40 - E. Hadidi: *Un cycle limite d'un système planaire perturbé contrôlé*
16h40-17h - A. Khellaf: *The Resolution of the Spectral Pollution the Generalized Spectrum method*
17h-17h20 - A. Hafdallah: *On the optimal control of systems with missing data*
17h20-18h10 - ***Short talks and Posters presentation***

Thursday 20 June 2019

Session Chairman : Moncef Aouadi

9h-9h50 - L. Tébou: *Simultaneous and indirect control of waves: some recent developments and open problems*
9h50-10h40 - Y. Kian: *Introduction au problème de Calderón*
10h40-11h10 - ***Coffee break***
11h10-12h - E. Soccorsi: *Introduction to Borg-Levinson Inverse Spectral Theory*
12h-14h30 - ***Lunch***

Session Chairman : Louis Tébou

14h30-15h05 - M. González-Burgos: *Sharp estimates of the one-dimensional boundary control cost for parabolic systems*
15h05-15h40 - C. Jammazi: *On the finite-time stabilization of some hyperbolic control systems by boundary feedback laws: Lyapunov approach*
15h40-16h - ***Coffee break and closing***

Courses

Yavar Kian
Aix Marseille Université, France.

"Introduction au problème de Calderòn"

L'objectif de ce cours sera de présenter le problème de Calderòn qui consiste à déterminer une conductivité apparaissant dans une équation elliptique á partir d'excitation et de mesures sur le bord du domaine. Nous étudierons ce problème en nous basant sur l'approche initiée par Sylvester et Uhlmann basée sur les solutions optique géométrique complexes.

Éric Soccorsi
Aix Marseille Université, France.

"Introduction to Borg-Levinson Inverse Spectral Theory"

The study of inverse spectral problems goes back to V.A. Ambarzumian [1] who investigated in 1929 the problem of determining the real potential V appearing in the Sturm–Liouville operator $A = -\partial_{xx} + V$, acting in $L^2(0,2\pi)$, from partial spectral data of A. He proved in [1] that $V = 0$ if and only if the spectrum of the periodic realization of A equals $\{k^2 \,; k \in \mathbb{N}\}$. For the same operator acting on $L^2(0,\pi)$, but endowed with homogeneous Dirichlet boundary conditions, G. Borg [2] and N. Levinson[8] established that while the Dirichlet spectrum $\{\lambda_k \,; k \in \mathbb{N}^*\}$ does not uniquely determine V, nevertheless assuming that $\varphi_k'(0) = 1$ for $k \geq 1$, additional spectral data, namely $\{\|\varphi_k\|_{L^2(0,\pi)} \,; k \in \mathbb{N}^*\}$ is needed, where $\{\varphi_k \,; k \in \mathbb{N}^*\}$ is an $L^2(0,\pi)$-orthogonal basis of eigenfunctions of A. I.M. Gel'fand and B.M. Levitan [6] proved that uniqueness is still valid upon substituting $\varphi_k'(\pi)$ for $\|\varphi_k\|_{L^2(0,\pi)}$ in the one-dimensional Borg and Levinson theorem.

In 1998, the case where Ω is a bounded domain of $\mathbb{R}^n$, $n \geq 2$, was treated by A. Nachman, J. Sylvester and G. Uhlmann [9], and by N.G. Novikov [10]. Inspired by [6], these authors proved that the boundary spectral data $\{(\lambda_k, \partial_\nu \varphi_k);\ k \in \mathbb{N}^*\}$, where $\partial_\nu = \frac{\partial}{\partial \nu}$ denotes the outward normal derivative on $\partial\Omega$, and (λ_k, φ_k) is the k^{th} eigenpair of A, uniquely determines the Dirichlet realization of the operator A. This result has been improved in several ways by various authors. H. Isozaki [7] extended the result of [9] when finitely many eigenpairs remain unknown. Recently, M. Choulli and P. Stefanov [5] proved stable determination of V from the asymptotic behaviour of $(\lambda_k, \partial_\nu \varphi_k)$ as $k \to \infty$. This result was improved and extended to infinitely extended domains in [4].

This course of three lectures is an introduction to the mathematical analysis of inverse spectral problems of Borg-Levinson type. More precisely, they are concerned with the uniqueness and the stability issues, in the inverse problem of determining the electric potential of the multidimensional Laplace operator by boundary spectral data.

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Louis Tébou
Florida International University, Miami, USA.

"Simultaneous and indirect control of waves: some recent developments and open problems"

The control of a single wave equation has known some tremendous amount of activity since the pioneering works of Russell in the sixties and seventies, and especially after the publication of Lions' monographs on controllability. However, the control of multi-component systems of wave equations can arguably be said to be at an early stage. In the coming lectures, I will discuss some of my recent works in this area, focusing on the notion of simultaneous control (using the same control scheme in all components of the system at hand), and indirect control (using a single control for a system consisting of two components). I will also discuss some problems that I deem important to investigate.

Invited speakers

Moncef Aouadi
Université de Carthage, Tunisia.
"Exact controllability for nonlinear thermoviscoelastic plate problem"

In this talk, we prove that after certain threshold time moment the nonlinear damped thermoviscoelastic plate is exactly controllable. We present a global well-posedness result for the nonlinear problem when the nonlinear functions are continuous and globally Lipschitz. Then we establish conditions on the physical constants of the plate to guarantee that the roots of the characteristic equation of the linear operator are simple. Subsequently, we find the necessary and sufficient conditions to guarantee the exponential decay of solutions. Then by a powerful spectral approach, we determine the optimal rate decay by the physical constants. The exact controllability of the linear and nonlinear problems is shown by proving that the corresponding controllability mappings are surjective.

Ahmed Bchatnia
Université de Tunis El Manar, Tunisia.
"Observability and stabilizability of the wave equation with moving boundary"

We deal with the one dimensional wave equation in a bounded interval with assigned moving interior point $a(t)$ and Dirichlet boundary conditions are specified. Here $a(t)$ is assumed to move slower than the light and periodically. Moreover a is continuous, piecewise linear with two independent parameters. Our major concern will be an observation problem which is based measuring, at each $t > 0$ of the transverse velocity at $a(t)$.

Manuel Gonzàlez-Burgos
Universidad de Sevilla, Spain.
"Sharp estimates of the one-dimensional boundary control cost for parabolic systems"

In this talk we will present new results on the cost of the boundary controllability of parabolic systems at time $T > 0$. In particular, we will study sharp estimates of the control cost at time T (T small enough) when the eigenvalues of the generator of the C_0 semigroup accumulate and do not satisfy a gap condition. The main ingredient we will use is the moment method.

Makram Hamouda
Université de Tunis El Manar, Tunisia.
"Theoretical and numerical aspects related to the energy of some Timoshenko systems"

In this talk, we consider a Timoshenko system subject to different types of dissipation (undamped, linear damping and nonlinear damping). First, we recall some theoretical asymptotic results related to the associated energy. Then, we design a discretization scheme, based on a combination of the finite element method and the finite difference one. The obtained scheme reaches to describe the discrete energy showing the positivity, the energy conservation property and the types of the decay rates in the case of a damped system. We numerically confirm the theoretical results established on the decay rates of the energy associated with each type of dissipation.

Chaker Jammazi
Université de Carthage, Tunisia.

"On the finite-time stabilization of some hyperbolic control systems by boundary feedback laws: Lyapunov approach"

This talk discusses the finite-time stabilization of some linear hyperbolic partial differential equations by means of persistently exciting boundary feedbacks. The construction of stabilizing feedbacks is based on Lyapunov approach, and the finite-time stability of the closed loop system is obtained by the persistent excitation condition of the boundary. A particular attention is paid to transport and wave control systems. The obtained results are used to solve two problems: the first one is finite-time stability of the SMB chromatography while the second application is the finite-time controllers of two strings connected by point mass.

Souleymane Khadri Harouna
Université de La Rochelle, France.

"A Stochastic Sub-grid Viscosity Model and Wavelet based Method for Images Assimilation"

The talk will concern the derivation of a stochastic model of the Navier-Stokes equations. We use a decomposition of the velocity field into a deterministic drift component and a random uncertainty term. The evolution equations of the deterministic component of the flow are derived from a stochastic version of the Reynolds Transport Theorem. These equations include in their general form an anisotropic diffusion term that represents the effect of unresolved turbulent scales modeled by stochastic uncertainty term. This is similar to the LES or RANS methods, in order to model the effects of the coarse component of a turbulent flow. Without forcing term, an analysis of the drift component equations allows to obtain physical properties such as the global energy dissipation similar to the classical Navier-Stokes equations. Then, some results on the existence of weak solutions can be obtained. Finally, we will present a wavelet based numerical discretization of the model and its application to variational image data assimilation.

Short Talks (20 mn)

"The universal bound property for a class of second order ODEs"

Mama Abdelli[1] and Alain Haraux[2]
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We consider the scalar second order ODE

$$u'' + |u'^{\alpha}|u' + |u|^{\beta}u = 0,$$

where α, β are two positive numbers, after the limited elementary approach of [3], Philippe Souplet [5] gave a definitive negative answer at least when $\alpha \geq \beta \geq 0$. Strangely enough, although this equation has been studied very carefully in [4] which several extensions in [1] and [2], it seems that nobody questionned the possibility of universal boundedness when $0 < \alpha < \beta$ and the non-linear semi-group $S(t)$ generated on $\mathbb{R}^2$ by the system in (u, u'). We prove that $S(t)\mathbb{R}^2$ is bounded for all $t > 0$ whenver $0 < \alpha < \beta$ and moreover there is a constant C independent of the initial data such that

$$\forall t > 0, \;\; u'^2(t) + |u|^{\beta+2} \leq C \max\{t^{-\frac{2}{\alpha}}, t^{-\frac{(\alpha+1)(\beta+2)}{\beta-\alpha}}\}.$$

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"A general decay result of a wave equation with a dynamic boundary control of diffusive type"

Abbes Benaissa[1], Hanane Benkhedda[2] and Farida Cheheb[3]
Laboratory of Analysis and Control of PDEs, Department of Mathematics, University of Sidi Bel Abbes, Algeria
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We study a wave equation with a dynamic boundary control of diffusive type. We establish optimal and explicit energy decay formula by using resolvent estimates. Our new result generalizes and improve the earlier related results in the literature.

"Permutating quantum eigenmodes by using the quasi-adiabatic motion of a wall"

Alessandro Duca, Romain Joly and Dmitry Turaev

The aim of the work is to study the Schrödinger equation $i\partial_t^2\psi = -\Delta\psi + V\psi$ on $L^2((0,1),\mathbb{C})$ when V is a localized potential wall with time-dependent position and intensity. In particular, we show how to control the eigenmodes of the equation with a suitable motion of the potential wall by following the ideas introduced in [1]. The result is achieved by defining a suitable dynamics of V which alternates adiabatic motion with fast accelerations in short time. Such accelerations are required in order to obtain the outcome and to avoid the tunnel effect intensifying when the wall reaches certain positions. In conclusion, we discuss further applications of the proposed techniques such as the validity of the global approximate controllability for such equation.

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"Un cycle limite d'un système planaire perturbé contrôlé"

Elbahi Hadidi[1] and Nadjet Stihi[2]
Univ Badji Mokhtar, Annaba - Faculté des Sciences - Département de Mathématiques & LaPS Laboratoire, Algeria
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Dans ce travail, nous étudions l'existence d'un cycle limite stable pour un type de système planaire perturbé contrôlé. Ensuite, nous avons consolidé cette étude par des exemples et des simulations avec Maple.

"On the optimal control of systems with missing data"

Abdelhak Hafdallah
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The aim of this talk is to present some basics and generalities about the optimal control of systems with incomplete data, the concept of no-regret control introduced by J.L.Lions to treat this kind of optimal control problems. The main idea of this method is to transfer an optimal control problem with missing data to a standard optimal control problem by using the concept of low-regret control. The no-regret control will be characterized by an optimality system independently of the missing data.

"Existence of solutions to Neumann problem involving multiple critical exponents"

Yamina Hamzaoui
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We consider the solvability of the Neumann problem for an elliptic system of two equations with weights involving two critical Sobolev exponents on a bounded domain in $\mathbb{R}^N$. By using variational methods, we investigate the effect of the shape of the graph of the weight functions and the geometry of the boundary on the existence of solutions.

"The Resolution of the Spectral Pollution the Generalized Spectrum method"

Ammar Khellaf[1] and Hamza Guebbai[2]
Laboratoire de Mathématiques Appliquées et de la Modélisation, Université 8 Mai 1945, Algéria
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It is well known that the spectral pollution is the weakness of projection methods applied on unbounded operator. This phenomenon is considered as a serious problem in several areas in the field of applied mathematics. Thus, our technic is an alternative method based on disrupting an unbounded operator by a bounded one until that the spectral properties transform to controlled case. The generalized spectrum takes its place as the desired case. We show that every unbounded operator contains a decomposition of two bounded operators which carry all the spectral properties.

Through this decomposition and basing upon its numerical approximations, the phenomenon of spectral pollution will be resolved.

This phenomenon is considered as a serious problem in several areas in the field of applied mathematics that has been studied.

The concept of generalized spectrum was constructed under the generalized spectrum of the matrices then extended for bounded operators case.

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"Global persistence of geometric structures for stratified Euler system with fractional dissipation"

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The present work deals with the stratified Euler system with fractional dissipation,

$$\begin{cases} \partial_t v + v\cdot\nabla v + \nabla p = \rho e_2, \quad \partial_t \rho + v\cdot\nabla\rho + (-\Delta)^{\frac{\alpha}{2}}\rho = 0 \quad (t,x)\in\mathbb{R}_+\times\mathbb{R}^2, \\ \operatorname{div} v = 0, \quad (v,\rho)_{|t=0} = (v_0,\rho_0). \end{cases} \tag{1}$$

Here, v refers to the velocity vector field in $\mathbb{R}^2$, the pressure p and the density ρ are two scalar functions. The buoyancy force ρe_2 models the effects of the gravity with $e_2=(0,1)$ and $(-\Delta)^{\frac{\alpha}{2}}$ designates the fractional laplacian with $\alpha\in]0,2]$. In $\mathbb{R}^2$, the vorticity ω of the velocity field v may be identified by $\omega \triangleq \partial_1 v^2 - \partial_2 v^1$. Consequently, the vorticity-density formulation of (1) is given by

$$\begin{cases} \partial_t \omega + v\cdot\nabla\omega = \partial_1\rho, \quad \partial_t\rho + v\cdot\nabla\rho + (-\Delta)^{\frac{\alpha}{2}}\rho = 0 \quad (t,x)\in\mathbb{R}_+\times\mathbb{R}^2, \\ (\omega,\rho)_{|t=0} = (\omega_0,\rho_0). \end{cases} \tag{2}$$

If $\rho \equiv \rho_0$, the system (1) is reduced to the classical $2d-$ incompressible Euler equations which are well-studied. In particular, when the initial vorticity having a vortex patch structure, that is to say, ω_0 is uniformly distributed over a bounde domain Ω_0 in the sense $\omega_0 = \mathbf{1}_{\Omega_0}$, Chemin has succeed in [2] to recover the Euler system globally in time. The cornerstone in his proof is the stationary logarithmic estimate. Afterwards, the Chemin's result was extended for several systems and differents regularities by various authors, see for instance [1,3,4,5,6,7,8,9,10].

The study of the vortex patch problem for (2) has been started recently in [9] where Hmidi and the author studied the case $\alpha = 2$ and showed if the boundary of $\partial\Omega_0$ is a Jordan curve of $C^{1+\varepsilon}$ regularity with $\varepsilon \in]0,1[$ then the velocity is Lipschitz for any positive time and the advected domain $\Omega_t \triangleq \Psi(t,\Omega_0)$ keeps its initial regularity. Moreover, we show that the vorticity $\omega(t) = \mathbf{1}_{\Omega_t} + \widetilde{\rho}(t)$, where $\widetilde{\rho}$ is a smooth function.

Here, we intend to lead the same result for the critical case $\alpha = 1$. Roughly speaking, we shall prove the persistence regularity of the initial patch and study the asymptotic behavior for the density.

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Short Talks (5mn).

"Regional controllability with minimum energy of wave equation"

Mouna Abdelli[1] and Abdelhak Hafdallah[2]
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In this paper, we have discussed the characterization of the control achieving regional gradient controllability with a minimum energy of systems described by wave equation and reach the desired state given only on a part of the system domain. The approach is based on an extension of the Hilbert uniqueness method.

"Modeling and Control of Quadrotor Transporting Cable-Suspended Load in the Longitudinal Plane"

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Modeling and position stabilization problems in the longitudinal plane (XGY) of a quadrotor helicopter with suspended load have been investigated. In the first part we present an efficient modeling of the quadrotor with cable suspended load with eight degrees of freedom. The formalism used is based on the Lagrange-d'Alembert approach. Then, in the longitudinal plane, by using iterative Backstepping techniques and Lyapunov theory, we succeed to stabilize the study system in the neighborhood of a desired attitude and position. Finally, simulation results are presented to prove the effectiveness of control laws.

"Differential inclusion and control theory"

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The goal of the book of abstracts is to use the differential inclusions to prove the existence of solutions to control and optimization problems " notably optimal control". For this purpose the following challenges will be investigated: topoligical propreties of set of solutions, selection of solution with given properties, evaluation of viability theory

"About covering mappings with values in the space with a reflexive binary relation"

Sarra Benarab[1] and Hamza Guebbai[2]
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The concept of orderly covers extend to mappings acting from an ordered space X into space Y with a reflexive binary relation. An assertion is obtained about the existence of a solution $x \in X$ of the equation $\Upsilon(x,x) = y$, where $y \in Y$, the mapping $\Upsilon : X^2 \to Y$ one by one from the arguments is a covering, and on the other - antitone. An example of a concrete equation satisfying the assumptions of the proved assertion, to which the known results are not applicable, since Y is not an ordered space.

"Hamiltonians spectrum in fermi resonance"

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This work reviews the Birkhoff-Gustavson normal form theorem (BGNF) near an equilibrium point of a quantum Hamiltonian. The BGNF process is thereafter used to investigate the spectrum of Schrodinger operator in the 1:2 resonances A computer program is proposed to compute the coefficient of the BGNF up to any order.

"On global existence and Asymptotic behaviour of solutions for an intgro-differential equation with strong damping"

Hocine Braiki
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In this talk we consider a nonlinear viscoelastic Petrovsky equation in a bounded domain with a strong damping nonlinear:

$$u_{tt}(x,\,t)\,+\triangle^2 u(x,\,t)\,-\int_0^t h(t\,-s)\triangle^2 u(x,\,s)ds\,-f(\triangle u_t(x,\,t))=0\;(P)$$

$$u(\mathrm{O})=u_0\;u_t(0)=u_1\;\forall x\in\Omega$$

$u=\dfrac{\partial u}{\partial x}=0$ *if* $x\in\partial\Omega$.

Tahamatani and M. Shahrouzi studied the equation

$$u_{tt}(x,\,t)+\triangle^2 u(x,\,t)-\int_0^t h(t\,-s)\triangle^2 u(x,\,s)ds=|u|^{p-2}u\;(1.1)$$

they prove the existence of weak solutions with initial boundary value conditions. Mean- while, they show that there are solutions under some conditions on initial data which blow up in finite time with non-positive initial energy as well as positive initial energy and give the lifespan estimates of solutions. In the absence of nonlinear source term, J.E. Munoz Rivera, E.C. Lapa and R. Baretto considered Eq. (1.1) in a bounded domain.

The main purpose of this paper is to prove global solvability and energy decay estimates of the solutions of problem (P) when h is of exponential decay rate. We would like to see the influence of frictional and viscoelastic damping on the strong dissipation. To obtain global solutions of problem (P), we use the Galerkin approximation scheme together with the energy estimate method.

To prove decay estimates, we use a perturbed energy method and some properties of convex functions. These arguments of convexity were introduced and developed by Cavalcanti et al. and Lasiecka and Tataru, and used by Liu and Zuazua and Alabau-Boussouira.

"Exact controllability of heat equation with control and white noise in the boundary"

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A parabolic stochastic system governed by a heat equation with a control and a white noise on the boundary is considered for the study of the exact controllability in Hilbert space. The exact controllability problem has been reduced to a moment problem for determining control.

"An exterior penalty method for nonlinear optimal control problems with mixed control-state constraints"

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The purpose of this talk, in the first part, is to discuss, via an exterior penalty functions method, a class of nonlinear optimal control problems with additional equality and inequality state and control constraints. Two different kinds of penalties are given, In the first, equality and inequality state-control constraints are penalized in a way that guarantees the exteriority of the approaching solutions. This property allows a sequence of optimal control problems (without constraints) and , under reasonable assumptions, we generate a sequence of minimizing points which converge to the solution of the original control problem. In the second, We use the penalty method in which the control and the state function are at the same level and the state equation becomes a general equality type constraint. The penalty function used, in this case, gives a sequence of truly unconstrained optimization problems. It turns out that the obtained sequence of minimizing points offer a minimizing point as well as a solution of the system differential equations (as quality constraints) for the original problems. This work is a continuation and refinement of the research treatment of the constrained control problems by the two different kinds of penalties. Note that this treatment yields certain useful properties to insure the strong compactness in the Lebesgue spaces and new techniques for studies of many problems in the literature. We treat here a fixed endpoint optimal control problem with specified; intermediate state and control constraints. We assume these constraints are given by finite systems of inequalities and equality constraints. In the second part, we give the necessary conditions of optimality of the solutions of the sequence of penalized problems in Hilbert case and under smoothness assumptions, we also obtain a sequence of maximum principle, and the fact that corresponding optimal controls provide minimizing sequence for the original constrained optimal control problem, we show that the sequence given by the Pontryagin's maximum approximate the Lagrange multipliers of the initial problem.

"About averaged controllability of systems depending on a parameter"

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In this talk we present the notion of average controllability which was introduced to control systems depending an parameter in finite and in infinite dimensional; also we present the average optimal control.

"Solutions for singular problem involving critical nonlinearity"

Fatiha Chouaou and Atika Matallah

This work deals with the existence and multiplicity of solutions to the following Kirchhoff problem with the critical exponent

$$(p_\lambda,\mu)\begin{cases} -div\left(\frac{|\nabla|^{p-2}\nabla u}{|x|^{pa}}\right)-\mu\frac{|u|^{p-2}u}{|x|^{p(a+1)}}=\frac{|u|^{p_*-2}u}{|x|^{p_*b}}\ \text{in}\,\Omega,\\ u=0\ \text{on}\ \partial\Omega,\end{cases}$$

where $1<p<N, 0\le a<\frac{N-p}{p}, a\le b<a+1, 0\le\mu<\bar{\mu}_a:=\left(\frac{N-p(a+1)}{p}\right)^p$, λ is a positive parameter, $p_*=\frac{pN}{N-p(a+1-b)}$ is the critical Caffarelli-Kohn-Nirenberg exponent, $f\in(W_a^{1,p}(\Omega))'\setminus\{0\}$. We prove that there exists $\Lambda>0$ such that problem $(p_{\lambda,\mu})$ has a nontrivial solution for any $\lambda\in(0,\Lambda)$.

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"About Arutyunov theorem of coincidence point for two mappings in metric spaces"

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In the famous theorem of Arutyunov, it is asserted that the mappings ψ, φ, acting from the complete metric space (X, ρ_X) to the metric space (Y, d_Y) , one of which is $\alpha-$covering and the second is β-Lipschitz, $\alpha > \beta$, have the coincidence point is the solution of the equation $\psi(x) = \varphi(x)$. We show that this assertion remains valid also in the case when the space Y is not metric it is sufficient that the function $d_Y : Y^2 \to \mathbb{R}^+$ satisfies only the axiom of identity. The function d_Y may not be symmetric and does not correspond to the triangle inequality; moreover, it does not have to satisfy the f -triangle inequality (that is, it is possible that the space Y is not even f-quasimetric).

"Distance from an Exactly Controllable System"

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A linear control system in the Hilbert space is considered. The distance to the uncontrollable is the distance between this system and the set of uncontrollable systems [i.e the distance from a controllable linear system to the nearest uncontrollable system.] A minor of this distance is determined.

"Etude de l'existence globale d'une équation non-linéaire de type Petrovsky"

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Cet exposé est consacré à l'étude de l'existence d'une solution globale d'une équation d'une plaque mince viscoelastique non-linéaire de type Petrovsky, en utilisant la méthode de Faedo-Galerkin.

"Punctual Sentinels for a geometrical problem"

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The aim of this paper is to estimate the shape of an unknown part of the boundary of a geometrical domain by the observation of the solution of a diffusion problem on a number of points in the inside of this domain. The technique used for this estimation is based on the sentinels theory. The interest of this work is to obtain an estimation result with a simple observation.

"On normality of Toeplitz matrice's power"

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A finite Toeplitz matrix T is normal if and only if it is a rotation and translation of a Hermitian Toeplitz matrix (type I) or is a generalised circulant (type II).In this paper we will study the structure of the powers of a matrix T of type (I) and (II)) (if it remains the same).

"Control of Fractional-order Mathieu-Duffing equation"

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In this study, we use the feedback control method to control fractional-order Mathieu-Duffing equation. The aim of this method is to add a control term to the system, in order to eliminate the chaotic behavior. The results are proved analytically by applying the stability condition for fractional system. Moreover numerical simulations are presented to verify the effectiveness of the proposed control scheme.

"General decay result of the timoshenko system in thermoelasticity of second sound with distributed delay"

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In this paper we consider one-dimensional linear thermoelastic system of Timoshenko type, where the heat fux is given by Cattaneo's law. We consider damping terms acting on the first and the second equation, as well as a distributed delay term on the first equation, It is known that an arbitrarily small delay may be the source of instability, we establish a general decay estimate where the exponential and polynomial decay rates are only particular cases. We establish our result without the usual assumption of the wave speeds. Our method of proof uses the energy method together with some properties of convex functions.

"Exponential decay of solutions of a nonlinearly damped wave equation"

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The issue of stablity of solutions to nonlinear wave equations has been addressed by many authors. So many results concerning energy decay have been established. Here in this paper we consider the following nonlin- early damped wave equation

$$u_{tt} - \triangle u + a(1+|u_t|^{m-2})u_t = bu|u|^{p-2},$$

$a,\ b > 0$, in a bounded domain and show that, for suitably chosen initial data, the energy of the solution decays exponentially even if $m > 2$.

"A New contributions in optimal control problems governed by singular systems via Balakrishnan's method"

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We study a class of optimal control problems (OCP) governed by regular linear differential-algebraic systems (DAEs). This class of optimal control problems can be transformed into problems of the calculus of variations via the Method of Balakrishnan. A convergence theorem is given to obtain approximate and, in the limit, an optimal solution of this class of optimal control problem by the use of the necessary optimality conditions of Euler-Lagrange type.

"Study of a problem for the biharmonic operator in a family of planar open sets"

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On the family of truncated sectors of the plane

$$\Omega_\omega = \{(r,\theta); 0 < r < 1, 0 < \theta < \omega\},\ 0 < \omega \leq \pi,$$

we study the family of problems $\Delta^2 u_\omega = f_\omega$ with boundary condition $u_\omega = \Delta u_\omega = 0$. There, the second members are assumed to depend smoothly on ω in $L^2(\Omega_\omega)$.
If $\omega < \pi$ it is known from [2] that the solution u_ω decomposes as

$$u_\omega = u_{1,\omega} + u_{2,\omega} + u_{3,\omega}, \tag{1}$$

where $u_{1,\omega}$, $u_{2,\omega}$ are singular and $u_{3,\omega}$ is regular. Indeed, near the origin, $u_{1,\omega}$ (resp. $u_{2,\omega}$, $u_{3,\omega}$) is of regularity $H^{1+\frac{\pi}{\omega}-\varepsilon}$ (resp. $H^{2+\frac{\pi}{\omega}-\varepsilon}$, H^4) for every $\varepsilon > 0$, while the solution u_π is, in the neighborhood of the origin again, of regularity H^4.
One clearly sees a resolution of the singularity near the angle π whose description is the main objective of this work. The obtained result is that there exists a decomposition (1) of u_ω which is uniform with respect to ω, when $\omega \to \pi$, with the best possible topologies for each term, and which converges towards the Taylor expansion of u_π near 0.

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Posters.

"Analysis of iterative learning control for a class of semilinear fractional differential equations"
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"Reconstruction of the surface deformation: The full Maxwell's equations"
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"Mathematical analysis of an HIV infection model"
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'On the Convergence of Moving Average Processes for Dependent Random Variables'
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"Overlapping nonmatching grid method for the ergodic control quasi variational inequalities"
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